Chapter 3

Triprasnadhikara - Three enquiries (Direction, Place and Time)

1. The structure of the material world
    a. Texts 9 – 12; procession of the equinoxes and calculation of the ayanamsa

8. Science
    a. Texts 1 – 8; construction of the vedic sundial and some general rules
    b. Texts 13 – 14; determination of latitude and co-latitude from the dial
    c. Texts 14 – 15; calculation of the Sun's zenith distance from the dial
    d. Texts 15 – 19; Further methods for calculating latitude, co-latitude, the equinoctial shadow, the Sun's declination and the tropical longitude of the Sun from the dial
    e. Texts 20 – 24; Calculation of the Sun's mean position, the Sun's meridian zenith distance, the shadow, the hypotenuse and the Sun's measure of amplitude.
    f. Texts 25 – 51; Other methods for calculating the same values, as well as calculation of more complex values, including the rising times of each sign and the position of the lagna.

Text & Translation

3.01a śilātale +ambusaṁśuddhe vajralepe +api vā same/
3.01b tatra śaṅkvaṅgulair iṣṭaiḥ samaṁ maṇḍalam ālikhet//

3.02a tanmadhye sthāpayec chaṅkuṁ kalpanādvādaśāṅgulam/
3.02b tacchāyāgraṁ spṛśed yatra vṛtte pūrvāparārdhayoḥ//

3.03a tatra bindū vidhāyobhau vṛtte pūrvāparābhidhau/
3.03b tanmadhye timinā rekhā kartavyā dakṣiṇottarā//

3.04a yāmyottaradiśor madhye timinā pūrvapaścimā/
3.04b diṅmadhyamatsyaiḥ saṁsādhyā vidiśas tadvad eva hi//

3.05a caturasraṁ bahiḥ kuryāt sūtrair madhyād vinirgataiḥ/
3.05b bhujasūtrāṅgulais tatra dattair iṣṭaprabhā smṛtā//

On a stony surface, made water level, or on hard plaster, made level, there draw an even circle, of a radius equal to any required number of the digits (angula) of the gnomon (sanku). At its centre set up the gnomon, of twelve digits of the measure fixed upon; and where the extremity of its shadow touches the circle in the former and after parts of the day, there fixing two points upon the circle, and calling them the forenoon and afternoon points, draw midway between them, by means of a fish figure (timi), a north and south line. Midway between the north and south directions draw, by means of a fish figure, an east and west line: and in like manner also, by fish figures (matsya), between the four cardinal directions, draw the intermediate directions. Draw a circumscribing square, by means of the lines going out from the centre; by the digits of its base line (bhujasutra) projected upon that is any given shadow reckoned.

3.06a prākpaścimāśritā rekhā procyate samamaṇḍale/
3.06b unmaṇḍale ca viṣuvanmaṇḍale parikīrtyate//

The east and west line is called the prime vertical (samamandala); it is likewise denominated the east and west hour circle (unmandala) and the equinoctial circle (vishuvanmandala).

3.07a rekhā prācyaparā sādhyā viṣuvadbhāgragā tathā/
3.07b iṣṭacchāyāviṣuvator madhyam agrābhidhīyate//

Draw likewise an east and west line through the extremity of the equinoctial shadow (vishuvadbha); the interval between any given shadow and the line of the equinoctial shadow is denominated the measure of amplitude (agra).

3.08a śaṅkucchāyākṛtiyuter mūlaṁ karṇo +asya vargataḥ/
3.08b projjhya śaṅkukṛtiṁ mūlaṁ chāyā śaṅkur viparyayāt//

The square root of the sum of the squares of the gnomon and shadow is the hypotenuse: if from the square of the latter the square of the gnomon be subtracted, the square root of the remainder is the shadow: the gnomon is found by the converse process.

3.09a triṁśatkṛtyo yuge bhānāṁ cakraṁ prāk parilambate/
3.09b tadguṇād bhūdinair bhaktād dyugaṇād yad avāpyate//

3.10a taddos trighnā daśāptāṁśā vijñeyā ayanābhidhāḥ/
3.10b tatsaṁskṛtād grahāt krānticchāyācaradalādikam//

3.11a sphuṭaṁ dṛktulyatāṁ gacched ayane viṣuvadvaye/
3.11b prāk cakraṁ calitaṁ hīne chāyārkāt karaṇāgate//

3.12a antarāṁśair athāvṛtya paścāc cheṣais tathādhike/
3.12b evaṁ viṣuvatī chāyā svadeśe yā dinārdhajā//

In an Age (yuga) the circle of asterisms (bha) falls back eastward thirty scores of revolution. Of the result obtained after multiplying the sum of days (dyugana) by this number, and dividing by the number of natural days in an Age, take the part which determines the sine, multiply it by three, and divide by ten; thus are found the degrees called those of the procession (ayana). From the longitude of a planet as corrected by these are to be calculated the declination, shadow, ascensional difference (caradala), etc. The circle, as thus corrected, accords with its observed place at the solstice (ayana) and at either equinox; it has moved eastward, when the longitude of the sun, as obtained by calculation, is less than that derived from the shadow, by the number of degrees of the difference; then, turning back, it has moved westward by the amount of difference, when the calculated longitude is greater. In like manner, the equatorial shadow which is cast at midday at one's place of observation.

Alternate translation:
In a cycle of four ages, the orbit of the Sun oscillates in the zodiac six hundred times. Of the result obtained after multiplying the sum of days (dyugana) by this number, and dividing by the number of natural days in an Age, take the part which determines the sine, multiply it by three, and divide by ten; thus are found the degrees called those of the procession (ayanamsa). From the longitude of a planet as corrected by this, is to be calculated its declination, shadow, ascensional difference (caradal), etc [as outlined in this chapter]; the results of such corrected calculations will agree with the observations at both the equinoxes and the solstices (or in the northern and southern movements as well as when on the equator). The orbit of the sun has moved eastward, by the difference of degrees, when the calculated longitude of the Sun made from its shadow is less [than that arrived at

by spasta calculations given in chapters 1 & 2]; then, turning back, it has moved westward by the amount of difference, when the longitude calculated from the shadow is greater. All of this is to be calculated from the equinoctial shadow which is cast at midday at one's place of observation.

3.13a dakṣiṇottararekhāyāṁ sā tatra viṣuvatprabhā//
3.13b śaṅkucchāyāhate trijye viṣuvatkarṇabhājite/

3.14a lambākṣajye tayoś cāpe lambākṣau dakṣiṇau sadā/
3.14b madhyacchāyā bhujas tena guṇitā tribhamaurvikā//

3.15a svakarṇāptā dhanurliptā natās tā dakṣiṇe bhuje/
3.15b uttarāś cottare yāmyās tāḥ sūryakrāntiliptikāḥ//

3.16a digbhede miśritāḥ sāmye viśliṣṭāś cākṣaliptikāḥ/
3.16b tābhyo +akṣajyā ca tadvargaṁ projjhya trijyākṛteḥ padam//

3.17a lambajyārkaguṇākṣajyā viṣuvadbhātha lambayā/
3.17b svākṣārkanatabhāgānāṁ diksāmye +antaram anyathā//

3.18a digbhede +apakramaḥ śeṣas tasya jyā trijyayā hatā/
3.18b paramāpakramajyāptā cāpaṁ meṣādigo raviḥ//

3.19a karkyādau projjhya cakrārdhāt tulādau bhārdhasaṁyutāt/
3.19b mṛgādau projjhya bhagaṇān madhyāhne +arkaḥ sphuṭo bhavet//

Upon the north and south line of the dial – that is the equinoctial shadow (vishuvatprabha) of that place – radius, multiplied respectively by gnomon and shadow, and divided by the equinoctial hypotenuse, gives the sines of co-latitude (lamba) and of latitude (aksha): the corresponding arcs are co-latitude and latitude, always south. The midday shadow is the base (bhuja); if radius be multiplied by that, and the product divided by the corresponding hypotenuse, the result, converted to arc, is the Sun's zenith distance (nata), in minutes: this, when the base is south, is north, and when the base is north, is south. Of the Sun's zenith distance, and his declination, in minutes, take the sum, when their direction is different – the difference, when it is the same; the result is the latitude in minutes. From this find the sine of latitude; subtract its square from the square of the radius, and the square root of the remainder is the sine of co-latitude. The sine of latitude, multiplied by twelve, and divided by the sine of co-latitude, gives the equinoctial shadow. The difference of the latitude of the place of observation and the Sun's meridian zenith distance in degrees (nata-bhagas), if their direction be the same, or their sum, if their direction be different, is the Sun's declination: if the sine of this latter be multiplied by radius and divided by the sine of greatest declination, the result, converted to arc, will be the Sun's [tropical] longitude, if he is in the quadrant commencing with Aries; if in that commencing with Cancer, subtract from a half circle if in that commencing with Libra, add a half circle; if in that commencing with Capricorn, subtract from a circle: the result, in each case, is the true [tropical] (sphuta) longitude of the Sun at midday.

3.20a tanmāndam asakṛd vāmaṁ phalaṁ madhyo divākaraḥ/
3.20b svākṣārkāpakramayutir diksāmye +antaram anyathā//

3.21a śeṣaṁ natāṁśāḥ sūryasya tadbāhujyā ca koṭijyā/
3.21b śaṅkumānāṅgulābhyaste bhujatrijye yathākramam//

3.22a koṭijyayā vibhajyāpte chāyākarṇāv ahardale/
3.22b krāntijyā viṣuvatkarṇaguṇāptā śaṅkujīvayā//

3.23a arkāgrā sveṣṭakarṇaghnī madhyakarṇoddhṛtā svakā/
3.23b viṣuvadbhāyutārkāgrā yāmye syād uttaro bhujaḥ//

3.24a viṣuvatyām viśodhyodaggole syād bāhur uttaraḥ/
3.24b viparyayād bhujo yāmyo bhavet prācyaparāntare//

To this if the previously calculated equation of the apsis be applied, with a contrary sign, the Sun's mean longitude will be found. The sum of the latitude of the place of observation and the Sun's declination, if their direction be the same, or, in the contrary case, their difference, is the Sun's meridian zenith distance (natamsas); of that find the base sine (bahujya) and the perpendicular sine (kotijya). If, then, the base sine and radius be multiplied respectively by the measure of the gnomon in digits, and divided by the perpendicular sine, the results are the shadow and hypotenuse at midday. The sine of declination, multiplied by the equinoctial hypotenuse, and divided by the gnomon sine (sankujiva), gives, when farther multiplied by the hypotenuse of any given shadow, and divided by radius (madhyakarna), the Sun's measure of amplitude (arkagra) corresponding to that shadow. The sum of the equinoctial shadow and the Sun's measure of amplitude (arkagra), when the Sun is in the southern hemisphere, is the base, north; When the Sun is in the northern hemisphere, the base is found, if north, by subtracting the measure of amplitude from the equinoctial shadow; if south, by a contrary process – according to the direction of the interval between the end of the shadow and the east and west axis.

3.25a mādhyāhniko bhujo nityaṁ chāyā mādhyāhnikī smṛtā/
3.25b lambākṣajīve viṣuvacchāyādvādaśasaṅguṇe//

3.26a krāntijyāpte tu tau karṇau samamaṇḍalage ravau/
3.26b saumyākṣonā yadā kāntiḥ syāt tadā dyudalaśravaḥ//

3.27a viṣuvacchāyayābhyastaḥ karṇo madhyāgrayoddhṛtaḥ/
3.27b svakrāntijyā trijīvāghnī lambajyāptāgramaurvikā//

The midday base is invariably the midday shadow. Multiply the sines of co-latitude and latitude respectively by the equinoctial shadow and by twelve, and divide by the sine of declination; the results are the hypotenuse when the Sun is on the prime vertical (samamandala). When north declination is less than the latitude, then the midday hypotenuse (srava), multiplied by the equinoctial shadow, and divided by the midday measure of amplitude (agra), is the hypotenuse. If the sine of declination of a given time be multiplied by radius and divided by the sine of co-latitude, the result is the sine of amplitude (agramaurvika).

3.28a sveṣṭakarṇahatā bhaktā trijyayāgrāṅgulādikā/
3.28b trijyāvargārdhato +agrajyāvargonād dvādaśāhatāt//

3.29a punar dvādaśanighnāc ca labhyate yat phalaṁ budhaiḥ/
3.29b śaṅkuvargārdhasaṁyuktaviṣuvadvargabhājitāt//

3.30a tadeva karaṇī nāma tāṁ pṛthak sthāpayed budhaḥ/
3.30b arkaghnī viṣuvacchāyāgrajyayā guṇitā tathā//

3.31a bhaktā phalākhyaṁ tadvargasaṁyuktakaraṇīpadam/
3.31b phalena hīnasaṁyuktaṁ dakṣiṇottaragolayoḥ//

3.32a yāmyayor vidiśoḥ śaṅkur evaṁ yāmyottare ravau/
3.32b paribhramati śaṅkos tu śaṅkur uttarayos tu saḥ//

And this, being farther multiplied by the hypotenuse of a given shadow at that time, and divided by the radius, gives the measure of amplitude (agra), in digits (angula), etc. If from half the square of radius the square of the sine of amplitude (agrajya) be subtracted, and the remainder multiplied by twelve, and again multiplied by twelve, and then farther divided by the square of the equinoctial shadow increased by half the square of the gnomon – the result obtained by the wise is called the 'surd' (karani): this let the wise man set down in two places. Then multiply the equinoctial shadow by twelve, and again by the sine of amplitude, and divide as before: the result is styled the 'fruit' (phala). Add its square to the 'surd', and take the square root of their sum; this, diminished and increased by the 'fruit', for the southern and northern hemispheres, is the sine of altitude (sanju) of the southern intermediate directions (vidis); and equally, whether the Sun's revolution take place to the south or to the north of the gnomon (sanku) – only, in the latter case, the sine of the altitude is that of the northern intermediate directions.

3.33a tattrijyāvargaviśleṣān mūlaṁ dṛgjyābhidhīyate/
3.33b svaśaṅkunā vibhajyāpte dṛktrijye dvādaśāhate//

3.34a chāyākarṇau tu koṇeṣu yathāsvaṁ deśakālayoḥ/
3.34b trijyodakcarajāyuktā yāmyāyāṁ tadvivarjitā//

3.35a antyā natotkramajyonā svahorātrārdhasaṅguṇā/
3.35b trijyābhaktā bhavec chedo lambajyāghno +atha bhājitaḥ//

3.36a tribhajyayā bhavec chaṅkus tadvargaṁ pariśodhayet/
3.36b trijyāvargāt padaṁ dṛgjyā chāyākarṇau tu pūrvavat//

The square root of the difference of the squares of that and of radius is styled the sine of zenith distance (dris). If, then, the sine of the zenith distance and radius be multiplied respectively by twelve, and divided by the sine of altitude, the results are the shadow and hypotenuse at the angles (kona), under the given circumstances of time and place. If radius be increased by the sine of ascensional difference (cara) when declination is north, or diminished by the same, when declination is south, the result is the day measure (antya); this, diminished by the versed sine (utkramajya) of the hour angle (nata), then multiplied by the day radius and divided by radius, is the "divisor" (cheda); the later, again, being multiplied by the sine of co-latitude (lamba), and divided by radius, gives the sine of altitude (sanku): subtract its sine from that of radius, and the square root of the remainder is the sine of the zenith distance (dris): the shadow and its hypotenuse are found as in the preceding process.

3.37a abhīṣṭacchāyayābhyastā trijyā tatkarṇabhājitā/
3.37b dṛgjyā tadvargasaṁśuddhāt trijyāvargāc ca yat padam//

3.38a śaṅkuḥ sa tribhajīvāghnaḥ svalambajyāvibhājitaḥ/
3.38b chedaḥ sa trijyayābhyastaḥ svāhorātrārdhabhājitaḥ//

3.39a unnatajyā tayā hīnā svāntyā śeṣasya kārmukam/
3.39b utkramajyābhir evaṁ syuḥ prākpaścārdhanatāsavaḥ//

If radius by multiplied by a given shadow, and divided by the corresponding hypotenuse, the result is the sine of zenith distance (dris): the square root of the difference between the square of that and the square of the radius is the sine of altitude (sanku); which, multiplied by radius and divided by

the sine of co-latitude (lamba), gives the "divisor" (cheda); multiply the later by radius, and divide by the radius of the diurnal circle, and the quotient is the sine of the Sun's distance from the horizon (unnata); this, then, being subtracted from the day-measure (antya), and the remainder turned into arc by means of the table of versed sines, the final result is the hour angle (nata), in respirations (asu), east or west.

3.40a iṣṭāgrāghnī tu lambajyā svakarṇāṅgulabhājitā/
3.40b krāntijyā sā trijīvāghnī paramāpakramoddhṛtā//

3.41a taccāpaṁ bhādikaṁ kṣetraṁ padais tatra bhavo raviḥ/
3.41b iṣṭe +ahni madhye prākpaścād dhṛte bāhutrayāntare//

3.42a matsyadvayāntarayutes trispṛksūtreṇa bhābhramaḥ/
3.42b tribhadyukarṇārdhaguṇāḥ svāhorātrārdhabhājitāḥ//

3.43a kramād ekadvitribhajyās taccāpāni pṛthak pṛthak/
3.43b svādho +adhaḥ pariśodhyātha meṣāl laṅkodayāsavaḥ//

3.44a khāgāṣṭayo +arthago +agaikāḥ śaratryaṅkahimāṁśavaḥ/
3.44b svadeśacarakhaṇḍonā bhavantīṣṭodayāsavaḥ//

3.45a vyastā vyastair yutāḥ svaiḥ svaiḥ karkaṭādyās tatas trayaḥ/
3.45b utkrameṇa ṣaḍevaite bhavantīṣṭās tulādayaḥ//

Multiply the sine of co-latitude by any given measure of amplitude (agra), and divide by the corresponding hypotenuse in digits; the result is the sine of declination. This, again, is to be multiplied by radius, and divided by the sine of greatest declination; the quotient, converted into arc, is, in signs, etc., the Sun's place in the quadrant; by means of the quadrants is then found the actual longitude of the Sun at that point. Upon a given day, the distance of three bases, at noon, in the forenoon, and in the afternoon, being laid off, from the point of intersection of the lines drawn between them by means of two fish figures (matsya), and with a radius touching the three points, is described the path of the shadow. Multiply by the day radius of three signs, and divide by their own respective day-radii, in succession the sines of one, of two, and of three signs; the quotients, converted into arc, being subtracted, each from the one following, give, beginning with Aries, the times of rising (udayasavas) at Lanka; namely sixteen hundred and seventy, seventeen hundred and ninety-five, and nineteen hundred and thirty-five respirations. And these, diminished each by its portion of ascensional difference (carakhanda), as calculated for a given place, are the times of rising at that place. Invert them, and add their own portions of ascensional differences inverted, and the sums are the three sines beginning with Cancer: and these same six, in inverse order, are the other six, commencing with Libra.

3.46a gatabhogyāsavaḥ kāryā bhāskarād iṣṭakālikāt/
3.46b svodayāsuhatā bhuktabhogyā bhaktāḥ khavahnibhiḥ//

3.47a abhīṣṭaghaṭikāsubhyo bhogyāsūn praviśodhayet/
3.47b tadvat tadeṣyalagnāsūn evaṁ yātāt tathotkramāt//

3.48a śeṣaṁ cet triṁśatābhyastam aśuddhena vibhājitam/
3.48b bhāgair yuktaṁ ca hīnaṁ ca tallagnaṁ kṣitije tadā// (C bhāgahīnaṁ ca yuktaṁ ca)

3.49a prākpaścān natanāḍībhis tasmāl laṅkodayāsubhiḥ/
3.49b bhānau kṣayadhane kṛtvā madhyalagnaṁ tadā bhavet//

From the longitude of the Sun at a given time are to be calculated the ascensional equivalents of the parts past and to come of the sign in which he is: they are equal to the number of degrees traversed and to be traversed, multiplied by the ascensional equivalent (udayasavas) of the sign, and divided by thirty; then, from the given time, reduced to respirations, subtract the equivalent, in respirations, of the part of the sign to come, and also the ascensional equivalents (lagnasavas) of the following signs, in succession – so likewise, subtract the equivalents of the part past, and of the signs past, in inverse order; if there be a remainder, multiply it by thirty and divide by the equivalent of the unsubtracted sign; add the quotient, in degrees, to the whole signs, or subtract it from them: the result is the point of the ecliptic (lagna) which is at that time upon the horizon (kshitija). So, from the east or west hour angle (nata) of the Sun, in nadis, having made a similar calculation, by means of the equivalents in right ascension (lankodayasavas), apply the result as an additive or subtractive equation to the Sun's longitude: the result is the point of the ecliptic then upon the meridian (madhyalagna).

3.50a bhogyāsūn ūnakasyātha bhuktāsūn adhikasya ca/
3.50b sampīṇḍyāntaralagnāsūn evaṁ syāt kālasādhanam//

3.51a sūryād ūne niśāśeṣe lagne +arkād adhike divā/
3.51b bhacakrārdhayutād bhānor adhikke +astamayāt param//

Add together the ascensional equivalents, in respiration, of the part of the sign to be traversed by the point having less longitude, of the part traversed by that having greater longitude, and of the intervening signs – thus is made the ascertainment of time (kalasadhana). When the longitude of the point of the ecliptic upon the horizon (lagna) is less than that of the Sun, the time is in the later part of the night; when greater, it is in the day-time; when greater than the longitude of the Sun increased by half a revolution, it is after sunset.